Trisection of the angle and other classical problems.
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We know that this problem has no solution.
We want to know how to solve it.
A.Strugatsky, B.Strugatsky. "Monday begins at Saturday"

The trisection or the dividing of the given angle into three congruent parts is one of three classical construction problems formulated in Ancient Greece. Two remaining problems are *the duplication of the cube*, i.e. the construction of the cube having the volume twice greater than the given cube, and *the quadrature of the disc*, i.e. the construction of the square with the area equal to the area of the given disc. In XIX century it was proven that it is impossible to solve these problems using only a compass and a ruler without partition points. In particular, the trisection of the angle $\varphi = \pi/3$ is equivalent to the solution of the cubic equation $x^3 - 3x - 1 = 0$, and its roots cannot be constructed by a compass and a ruler, because it has no rational roots. But the problem can be solved, if we enlarge the list of allowed instruments, or if some curve is preliminary drawn on the plane. Below are two the most known problems.

1 Divide the given angle into three congruent parts using a compass and a ruler with two partition points bounding a unit segment.

Definition. Let a point move uniformly along a ray rotating with constant angle velocity around its origin. Then its trajectory is called *the Archimedes spiral*.

2 Let an Archimedes spiral be drawn on the plane. Using a compass and a ruler divide the given angle

a) into three congruent parts;

b) into n congruent parts.

The set of curves which make the trisection resolved is very great. The most interesting are the trisection methods using the conics. Since many graphic editors, for example Geogebra, contain a possibility to draw conics, it is possible to create an instrument of trisection using them.

1 Trisection by conics

All constructions of this section have to be performed using a compass and a ruler.

3 a) Let the parabola $y = x^2$ be drawn on the plane. A circle centered at point (a, b) passes through the vertex of the parabola. Write an equation having the roots coinciding with the X-coordinates of the remaining common points of the circle and the parabola.

b) Using a parabola drawn on the plane, divide the given angle into three equal parts.

c) Using a parabola, solve the problem of duplication of the cube.

4 a) The hyperbola $xy = C$ is drawn on the plane. A circle with center A lying on the hyperbola passes through the reflection of A about the center of the hyperbola. Prove that three remaining common points of the hyperbola and the circle are the vertices of a regular triangle.

b) Using an equilateral hyperbola drawn on the plane, divide the given angle into three congruent parts.

5 a) The angle between the asymptotes of a hyperbola drawn on the plane equals $2\pi/3$. A circle passes through the vertex of one branch of the hyperbola and the focus of the second branch. Prove that three remaining common points of the hyperbola and the circle are the vertices of a regular triangle.

b) Using the hyperbola with eccentricity 2 drawn on the plane, divide the given angle into three congruent parts.

6★ Try to create a method of trisection using come conic distinct from described above.